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Hierarchical social networks and information flow

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Abstract

Using a simple model for the information flow on social networks, we show that the traditional hierarchical topologies frequently used by companies and organizations, are poorly designed in terms of efficiency. Moreover, we prove that this type of structures are the result of the individual aim of monopolizing as much information as possible within the network. As the information is an appropriate measurement of centrality, we conclude that this kind of topology is so attractive for leaders, because the global influence each actor has within the network is completely determined by the hierarchical level occupied.

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1. Introduction

The study and characterization of complex systems is a very fruitful research field, where many interesting problems are still open. In particular, the study of complex networks, where graph and network analysis is applied, is becoming a popular topic

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of research with applications in different scientific and technological disciplines such as neurobiology [1–4], the Internet [5], the financial markets [6], etc. Moreover, many physicists have focused their research interests on complex networks, as it can be noticed by the number of papers appeared in the physics literature [7–10] in recent years.

Among complex networks, social networks appear in a quite natural way, playing an important role. Furthermore, as any other complex system, social networks are susceptible to be analyzed in the framework of graph theory [11]. There are many recent publications that show the power of graph theory methods for social network analysis in different fields like collaboration networks [12], growth of social networks [13], ego-centered networks [14], just to cite a few.

In the context of graph theory, a graph G consists basically on a nonempty set of elements, called vertices, and a list of unordered pairs of these elements, called edges. If i and j are vertices of G, then an edge of the form (i,j) is said to connect i and j. The number of vertices N in G is called the order of the graph, the number of edges connected to a particular vertex is called the degree of the vertex c and the number of edges in G divided by its size is called the average degree of the graph $\bar{c}$.

Graphs may be used to represent the relational structure of social networks in a quite natural way. Considering an organization, we call an actor to any of the individuals belonging to it. Each of the actors may be represented by a vertex of a graph. These actors usually maintain collaboration relationships among them, which may be represented as edges on the graph. For simplicity, the relations are assumed to be symmetric, so that the edges are always undirected. With this simple abstraction, it can be easily shown that the relational structure of a particular social community can be easily derived from its associated graph.

Using these basic ingredients, it is possible to model any type of social interaction. In this paper, our interest is focused on understanding how the information flows on the networks among the different actors. Some recent work on this field [15] has shown that it is possible to define simple models for the information transfer on social communities. Our objective is to use these models to demonstrate that there exists a close relationship between the structure and the information efficiency of organizations. Moreover, we aim to analyze the most common graph topologies and compare them, to conclude that the traditional hierarchical topologies commonly used by organizations are poorly designed in terms of efficiency.

The paper is organized as follows: we start analyzing how information and structure are related, then we dig into the properties of hierarchical regular trees, showing that hierarchical topologies only benefit the higher levels of the hierarchies. Finally, a comparison between different topologies is carried out and some interesting conclusions are extracted.

2. Information and structure in organizations

As is can be seen in Ref. [15], the efficiency of an organization, in terms of information, is closely related with the topology of the network modeling it. The simple model proposed there has been developed considering that the information travels preferently following the shortest paths and that there is certain degradation of the information in the traveling process. This model is based on a new magnitude measuring the information flow called the *coordination degree*. Given a graph G and given two members of the graph i and j, the *coordination degree* between them is defined as

$$\gamma_{ij} = \mathrm{e}^{-\xi d_{ij}} , \tag{1}$$

where d_{ij} is the distance between the two vertices and ξ is a real positive constant measuring the strength of the relationship, which we call the *coordination strength*. This means that the ability of two vertices to exchange information decays exponentially with the distance between them.

In the same sense, we define the *total coordination degree* for vertex i as the sum of all the coordination degrees of that particular vertex with the rest

$$\Gamma_i = \sum_{j=1}^{N} \gamma_{ij} , \tag{2}$$

where N is the order of the graph. Finally, the *average coordination degree*, $\bar{\Gamma}$, of the graph is defined as the mean of the coordination degrees of all vertices in the graph

$$\bar{\Gamma} = \frac{\sum_{i=1}^{N} \Gamma_i}{N} . \tag{3}$$

It is interesting to note that the total coordination degree of a vertex measures the amount of information managed by that vertex. So, it deals with a local property associated with a particular position on the network. On the other hand, the average coordination degree is a magnitude related to the total information traveling within the network and it can be seen as a measurement of the efficiency of the organization in terms of information transfer.

The important fact at this point is to note that the average coordination degree depends strongly on the structure of the graph which is analyzed. To show this, we may evaluate this parameter for some of the usual graph topologies. For example, as it can be seen in Ref. [15], in regular 2D lattices of degree $c = 4$

$$\bar{\Gamma} = 1 + 4 \sum_{j=1}^{n} j \mathrm{e}^{-\mathrm{j}\xi} , \tag{4}$$

where $2n^2 + 2n + 1 = N$ and N is the order of the graph. Nevertheless, if we choose a perfectly expanding graph topology of degree c, the average coordination degree takes a quite different expression

$$\bar{\Gamma} = 1 + c\mathrm{e}^{-\xi} \sum_{j=0}^{m} (c-1)^{\mathrm{j}} \mathrm{e}^{-\mathrm{j}\xi} , \tag{5}$$

where m is the number of levels of the graph verifying $N = 1 + c((c-1)^{m+1} - 1)/(c-2)$.

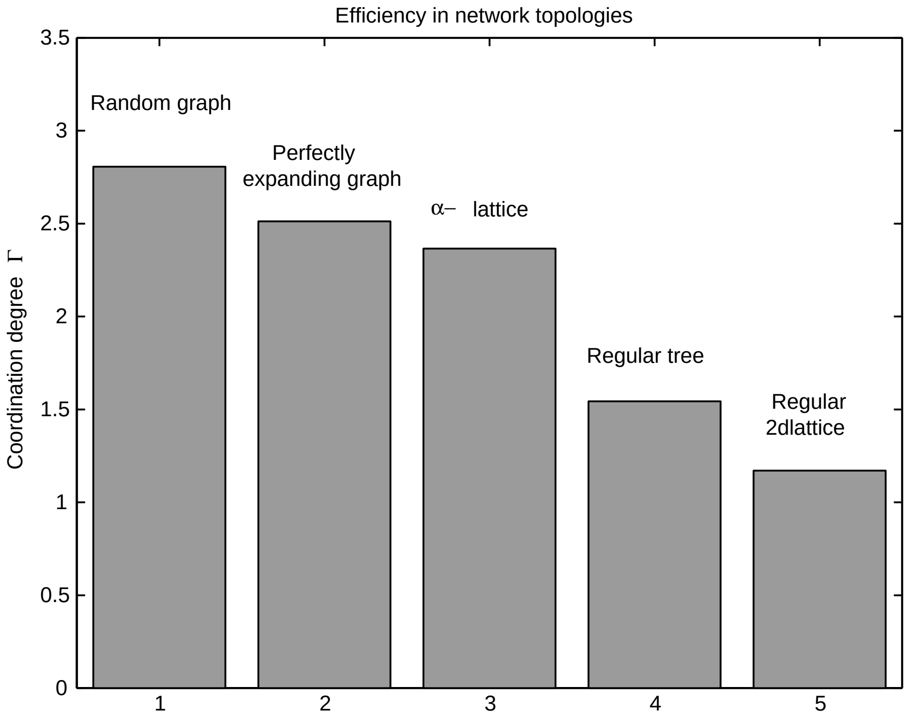

Fig. 1. Efficiency of different network topologies in terms of the average coordination degree. All graphs share the same number of actors, $N = 485$, and the same degree, $c = 4$.

As it can be understood, the average coordination degree may take very different values in a given organization depending on the topological structure of the relationships among its members. So, we must accept that this structure has a strong impact on the global efficiency of the networks. Moreover, it is reasonable to imagine that the topology should be designed in a way which maximizes this parameter, mainly in information-based organizations. When observing the structure of common organizations, it is easy to remark that most commercial companies, government organizations, the army and even universities usually follow strictly hierarchical rules in the design of their topologies. One could think that this fact may be related with an aim of maximizing their efficiencies. However, this is far from being true. Some simple simulations depicted in Fig. 1 show that regular trees are one of the worst topologies in terms of information transfer. This fact suggests that there might be reasons stronger than efficiency playing some crucial roles for these particular types of social networks. One reasonable explanation for this is that those social communities are mainly based on authority relations. So, hierarchical structures are prefered because they clearly reflect the position each actor occupies in the organization. In the following section, we show that hierarchical topologies indeed reflect this authority connections, benefitting the hierarchical levels in terms of information transfer.

3. Efficiency and information in hierarchical networks

In this section, we are particularly interested in the analysis of social networks having hierarchical topologies. The most characteristic example of graphs having this structure are regular trees. A regular tree is a regular graph (all vertices have the same degree c) that is connected (there is a path joining any two of its vertices) and that contains no circuits (there is no path going from one actor to itself that does not visit the same vertex twice). Every regular tree has a particular vertex, called root node or top of the tree, that is the most central vertex in the graph. It is important to note that, by definition, a regular tree must have an infinite order. To be able to deal with finite graphs, given a regular tree, we define its truncated regular tree of depth Λ as the $\Lambda - 1$ neighborhood of the root node (the subgraph composed by all its neighbors of order less or equal to $\Lambda - 1$ and the edges interconnecting them).

Fig. 2 shows a truncated regular tree with $\Lambda = 3$ and $c = 4$. It is easy to remark that this tree is not really regular because the second neighbors of the root node do not have c edges. Nevertheless, for clarity, in the rest of this paper we avoid using the term truncated and we talk directly about regular trees of depth Λ and degree c to refer to its truncated form.

Our objective in this section is to develop a mathematical framework for the study of the information flow, in terms of the coordination degree, in regular hierarchical trees representing social networks. With this purpose in mind, we follow a strategy consisting of two steps: first we investigate how the information flows from the bottom to the top of the tree, then we evaluate the coordination degree for each vertex in the opposite direction, starting at the top and finishing at the bottom. Finally, we show that this technique drives to a recursive expression which can be used to calculate the coordination degree for any actor in the network. Before starting, it is necessary to define a few more parameters.

We define the down level δ of a vertex in a regular tree as the level it occupies in the hierarchy starting the count at the bottom of the tree. For example, in a tree with 6 levels ($\Lambda = 6$), δ takes values between 1, for the nodes in the lowest hierarchical level, and 6, for the root vertex.

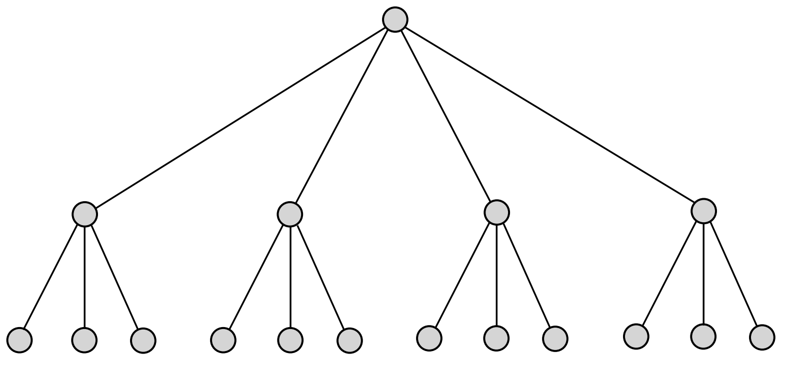

Fig. 2. A truncated regular tree with depth $\Lambda = 3$ and degree $c = 4$.

We define the up level υ of a vertex in a regular tree as the level it occupies in the hierarchy starting the count at the top of the tree. So, in the preceding example, υ varies between 6, for the lowest nodes, and 1, for the top. It is interesting to remark that given a regular tree with depth Λ, a vertex with down level δ must have $\upsilon = \Lambda - \delta + 1$ and vice versa.

Now, we define the exported information Υ of a vertex as the coordination degree of that vertex in the subgraph composed by all its neighbors located under that particular vertex in the graph (all the nodes with lower down level having a path to that vertex in this subgraph). In regular trees, all vertices with the same δ have the same exported information, so we use Υ_δ to denote the exported information of any vertex with down level δ.

It can be easily understood that $\Upsilon_1 = 1$, because all nodes with $\delta = 1$ have no neighbors with lower down level. Going up in the tree, we may see that $\Upsilon_2 = 1 + (c-1)e^{-\xi}$ (its own contribution plus the contribution of its $c-1$ order 1 lower neighbors) and $\Upsilon_3 = 1 + (c-1)e^{-\xi}(1 + (c-1)e^{-\xi})$. Note that this could be written in a more compact form in the following way: $\Upsilon_2 = 1 + (c-1)e^{-\xi}\Upsilon_1$, $\Upsilon_3 = 1 + (c-1)e^{-\xi}\Upsilon_2$. In fact, it can be proven by induction, that this relation remains true for all down levels in a regular hierarchical tree. So we may obtain a general recursive expression for the exported information in the following way:

$$\begin{cases} \Upsilon_i = 1 & \text{if } i = 1\,, \\ \Upsilon_i = 1 + (c-1)e^{-\xi}\Upsilon_{i-1} & \text{if } i > 1\,. \end{cases} \tag{6}$$

From these recursive equations one can obtain, $\Upsilon_i = (1 - y^i)/(1 - y)$ where $y = (c-1)e^{-\xi}$, and with the expression of the exported information, we may calculate the coordination degree each vertex obtains. We only need to consider a regular tree with depth Λ and degree c. The symmetry of the tree makes it easy to understand that the coordination degree for all vertices occupying the same up levels in the hierarchy is equal. So, we call Γ_υ the coordination degree for all vertices at up level υ. For regular trees, it may be proven that the information a node receives from any of their lower edges is proportional to the information that the vertex at the other side of that edge exports. The reason for this to be true is that, for the particular structure of a regular tree, there exists only one node-independent path joining any two pairs of nodes, so, all the vertices that can be attained through an edge are only reachable through it.

With these premises, it is straightforward to demonstrate that the coordination degree for the top vertex of the tree takes a value $\Gamma_1 = 1 + ce^{-\xi}\Upsilon_{\Lambda-1}$, falling one step in the hierarchy, we may observe that $\Gamma_2 = 1 + (c-1)e^{-\xi}\Upsilon_{\Lambda-2} + e^{-\xi} + (c-1)e^{-2\xi}\Upsilon_{\Lambda-1}$, which can be written in a more compact form as $\Gamma_2 = 1 + (c-1)e^{-\xi}\Upsilon_{\Lambda-2} + e^{-\xi}(\Gamma_1 - e^{-\xi}\Upsilon_{\Lambda-1})$. This expression takes into account the own contribution of the vertex, 1, the contribution that arrives from its lower edges, $(c-1)e^{-\xi}\Upsilon_{\Lambda-2}$, and the contribution obtained through its upper edge, $e^{-\xi}(\Gamma_1 - e^{-\xi}\Upsilon_{\Lambda-1})$. It may be easily understood that this relation can be generalized for all intermediate levels of the tree in the following way: $\Gamma_i = 1 + (c-1)e^{-\xi}\Upsilon_{\Lambda-i} + e^{-\xi}(\Gamma_{i-1} - e^{-\xi}\Upsilon_{\Lambda-i+1})$. However, for the nodes at the bottom of the tree there are no lower edges, so the expression obtained for them is $\Gamma_\Lambda = 1 + e^{-\xi}(\Gamma_{\Lambda-1} - e^{-\xi})$. These relations may be compacted into a single recursive expression of the form

$$\begin{cases} \Gamma_i = 1 + c\mathrm{e}^{-\xi}\Upsilon_{\Lambda-i} & \text{if } i = 1\,, \\ \Gamma_i = 1 + (c-1)\mathrm{e}^{-\xi}\Upsilon_{\Lambda-i} + \mathrm{e}^{-\xi}(\Gamma_{i-1} - \mathrm{e}^{-\xi}\Upsilon_{\Lambda-i+1}) & \text{if } 1 < i < \Lambda\,, \\ \Gamma_i = 1 + \mathrm{e}^{-\xi}(\Gamma_{\Lambda-1} - \mathrm{e}^{-\xi}) & \text{if } i = \Lambda\,. \end{cases} \tag{7}$$

Using these equations together with the solution of Eq. (6), one can evaluate analytically the information each actor is able to embrace within an organization structured as a regular tree, Γ_i. Due to these results, it is possible to analyze how the information is distributed within hierarchical structures, which is described in the following section.

4. Information distribution and hierarchical growth

As we have seen previously, hierarchical topologies are suboptimal in terms of efficiency. Knowing this, one interesting open question is determining why many leaders of companies and organizations choose this particular kind of topology. As hierarchical networks are usually based on authority relationships among the actors, showing that there exists a close connection between the hierarchical level of an actor and the information it receives, could be a successful justification suggesting why this kind of structure is so common. To prove this fact, the average coordination degree is not critical any more, because it is related with a global network parameter. Our interest must be concentrated on investigating how the information is distributed among the different actors of that particular community. To observe the effect of the hierarchical topology over the information distribution, it would be interesting to see how this information is shared among the vertices on other network structures. For clarity, we have chosen the most typical ones: regular lattices and random graphs.

Fig. 3 represents the probability distribution of the coordination degree in a typical random graph. It can be seen that most of the vertices are located near the maximum of the gamma shaped distribution and share the same amount of information, while only a minority receives manifestly a different amount of information than the rest. These fluctuations are mainly produced by the distribution of the degree itself, that it is known to follow a Poissonian law in random graphs [16]. This result means that some actors of the network receive more information, but with the cost of increasing the number of relations they maintain, and thus increasing the time and effort needed in developing their social interactions.

When considering regular lattices, the analysis is much more simpler. For infinite or periodic regular lattices, all vertices play the same role in the network and thus all share the same coordination degree. Thus, the probability distribution for the information is a Dirac-delta function centered in the particular value of the average coordination degree. This means that, in terms of information, regular lattices are the most democratic structures, because all actors in the community receive the same amount of information.

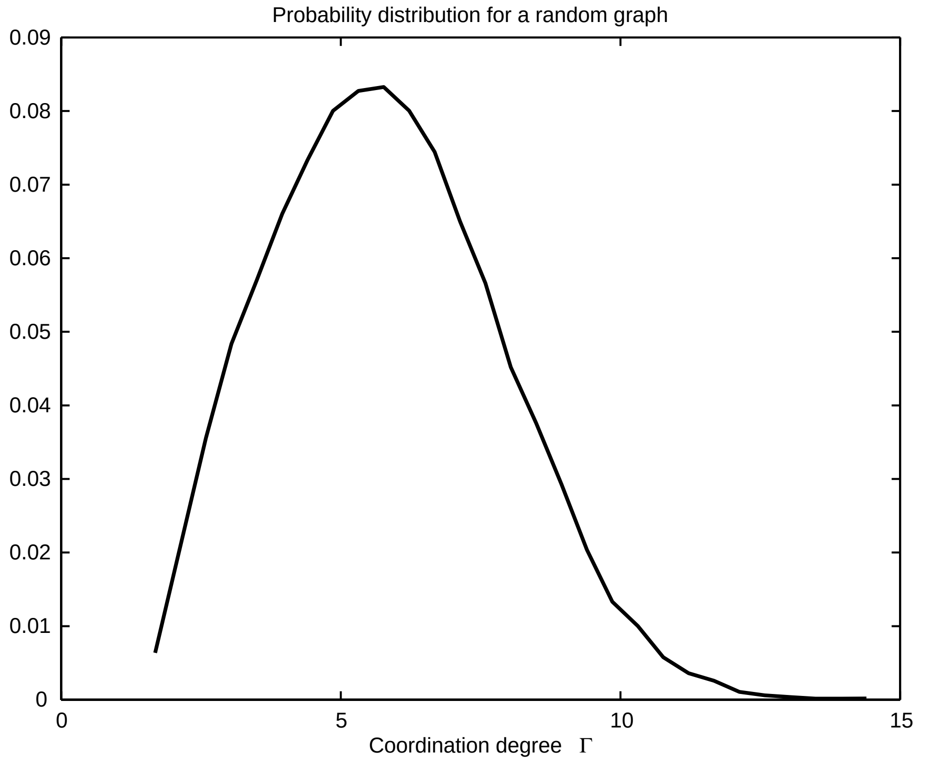

Fig. 3. The probability distribution of the coordination degree in a random graph of order $N = 200$ and average degree $\bar{c} = 3$. The figure has been averaged over 150 different graphs. Isolated vertices have been eliminated.

However, hierarchical trees have a particular and interesting behavior. As is can be derived from Eq. (7), the distribution of the coordination degree of the vertices follows a monotonic shape as depicted in Fig. 4. It can be observed that the number of nodes which share a particular value of the coordination degree decreases with it. Opposite to the behavior of random graphs, vertices receiving more information do not own a higher number of edges because, in regular trees, all nodes have the same degree (for truncated regular trees this is true except for the vertices located at the lowest hierarchical level).

Although nothing has been said about the location of the privileged vertices, one may imagine that this has a relation with the position they occupy in the hierarchy. Indeed, as Fig. 5 shows, the coordination degree of the vertices of the tree depends exclusively on their up level. It can be seen that, the higher the position on the hierarchy, the larger the amount of information that can be handled. This is indeed a very interesting conclusion because it shows that the topology of this kind of networks intrinsically benefits the nodes located in the upper levels of the hierarchy, allowing them to receive more information per edge than all the other vertices located under them.

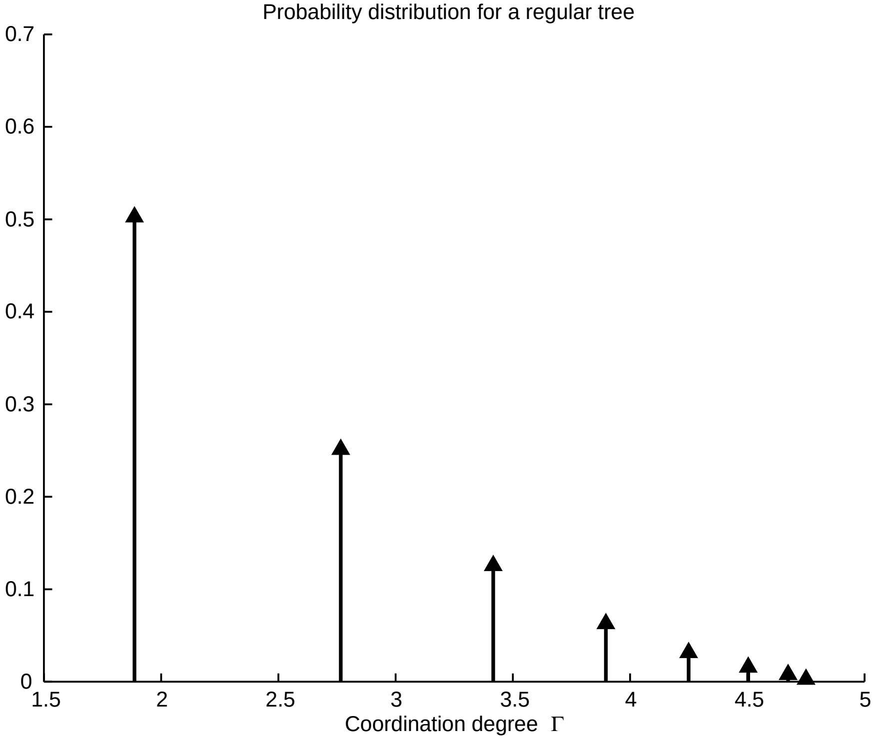

Fig. 4. Probability distribution of the coordination degree in a truncated regular tree of depth $\Lambda = 8$ and degree $c = 3$. The coordination strength is fixed to a value $\xi = 1.0$.

In fact, it can be demonstrated that this is a very general conclusion for regular trees. Independently on the number of relations c of each vertex, and on the strength of the relationships ξ, given two nodes with up levels $\upsilon_1 \leqslant \upsilon_2$, in any case $\Gamma_{\upsilon 1} \geqslant \Gamma_{\upsilon 2}$. This means that the hierarchical structure guarantees that, in terms of information, the efficiency of the relationships always improves when moving up in the tree. Thus, considering that all actors invest the same effort in relating with the others, the position on the hierarchy manifestly determines the amount of information that can be managed.

These ideas may justify why most organizations are hierarchical. In order to clarify this idea, we can develop a simple model of growth in social communities. For any type of organization, it is reasonable to assume that actors are able to spend only a limited time relating with the rest. That is, the number of relations they may maintain is limited to a fixed number c (the degree of each vertex is between 0 and c). Moreover, we accept that all actors make the same effort to maintain the relationship, that is, the strength of the relationships ξ is the same for all of them. For the growth model to be suitable, we must also assume two more rules. First, at the beginning, the social network is composed of a single actor and the growth takes place by the

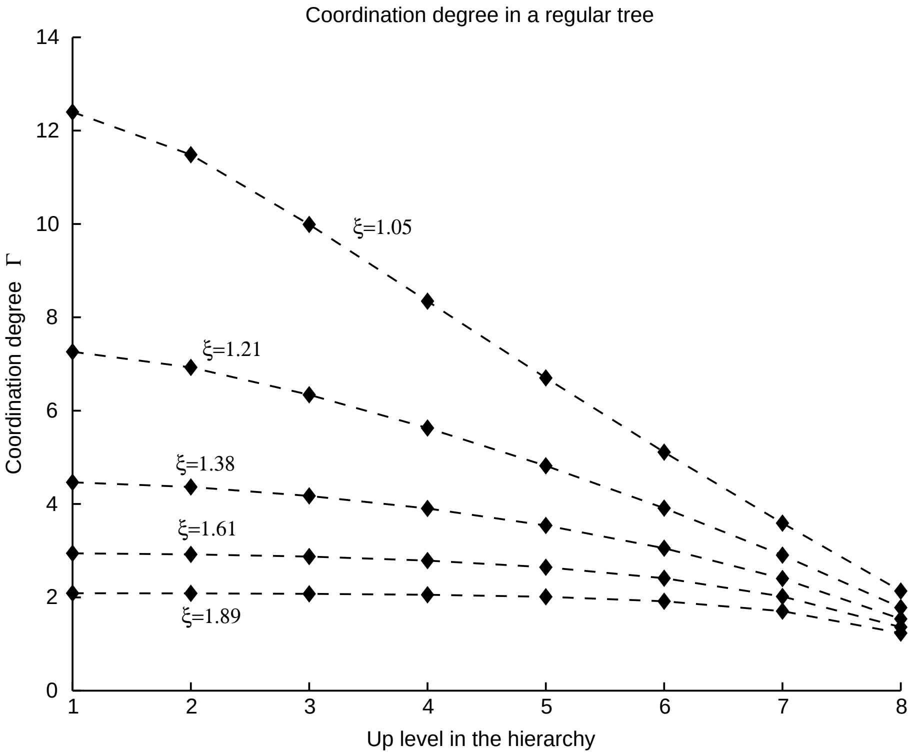

Fig. 5. Coordination degree of vertices as a function of the up level in the hierarchy. The simulation has been performed with a truncated regular tree of depth $\Lambda = 8$ and degree $c = 4$ for 5 different values of the coordination strength.

addition of new ones. Second, all new vertices added to the network can only establish a single relation with any of the preexisting members. Furthermore, the edge representing this relation must be established obtaining a maximum coordination degree (information) from the network. With these simple premises, it is easy to demonstrate that this network grows in a hierarchical tree topology, because all new vertices try to attach to the highest hierarchical level having at least one free relationship (a degree less than c).

This is a remarkable conclusion because it provides an explanation for the fact that most companies and organizations grow following strictly hierarchical structures and also illustrate why usually the higher levels of the hierarchy are occupied by the olders members. However, for this model to be plausible, it would be interesting to justify why new vertices wish to establish their single edge maximizing the coordination degree. In the following section, we show that the coordination degree is indeed an appropriate parameter for the measurement of the centrality (influence) an actor has within an organization.

5. Centrality and dominance in organizations

Centrality measures in social networks are an attempt to evaluate the social power or influence each actor has within a community. There are many different alternatives to measure this parameter, but certainly the most popular ones are closeness centrality, C_c, and betweenness centrality, C_b [17]. Closeness centrality is defined in terms of distance between nodes of the graph. Given a vertex v

$$C_c(v) = \frac{1}{\sum_{i \in G} d_{vi}} , \tag{8}$$

where G is the set of all vertices in the graph and d_{vi} is the distance between nodes v and i, while betweenness centrality is defined in terms of the number of shortest paths that pass through a given vertex v

$$C_b = \frac{1}{\sum_{i,j \in G} b_v(i,j)} , \tag{9}$$

where G is again the set of all vertices in the graph and $b_v(i,j)$ is 1 if the shortest path between i and j passes through v and 0 otherwise.

It is important to remark that both definitions of centrality maintain a close relation with the notion of information flow in the social network. According to Ref. [12], C_b is a measure of the control an actor has over the information flowing between others, while C_c is a measure of the influence of the vertex in terms of its access to the information. Indeed, considering the average distance a vertex maintains with all the others in a connected graph is an appropriate measure of the information it can reach. Nevertheless, it does not take into consideration other important parameters as the strength of the relation. For this reason, we propose the coordination degree as an alternative measurement of centrality in terms of the information a vertex can reach. In fact, it is easy to prove that both measurements maintain a very close relation: given two nodes u and v in a graph, it can be easily demonstrated that if $C_c(u) > C_c(v)$ then, for any value of the coordination strength $\xi > 0$, $\Gamma_u > \Gamma_v$ (note that if $\sum_{i \in G} d_{ui} < \sum_{i \in G} d_{vi}$ then, for all $\xi > 0$, $\sum_{i \in G} e^{-\xi d_{ui}} > \sum_{i \in G} e^{-\xi d_{vi}}$). As a measurement of centrality, the coordination degree provides the influence each actor has in a social community, so actors entering into a social network may try to attach to the network maximizing this parameter, as we assumed in the growth model proposed in the preceding section.

Although the coordination degree is a good approximation to the global influence a vertex has in the network, it is not appropriate to measure the supremacy the vertex has over the others. In hierarchical networks, where the relationships are established through authority connections, it would be interesting to define a new parameter evaluating the supremacy. A more suitable alternative to measure authority, in terms of information control between actors, is to evaluate the relative difference of information between the two vertices. Thus, given two vertices v and u on a graph, we define the domination of information of v with respect to u as in the graph as

$$\Delta_{uv} = \frac{\Gamma_v - \Gamma_u}{\Gamma_u} . \tag{10}$$

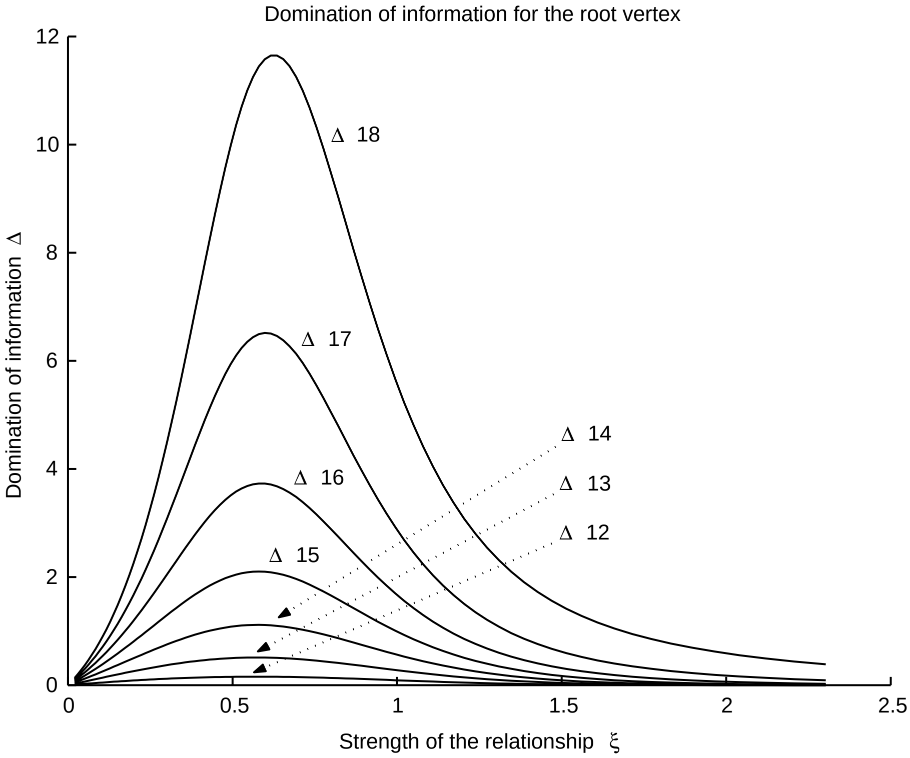

Fig. 6. Domination of information of the root vertex in a truncated regular tree of depth $\Lambda = 8$ and degree $c = 4$. It can be observed that the domination takes a maximum for a value of ξ over 0.5, which can be considered to be a high coordination strength.

The dominance of information can be used to observe another interesting feature of hierarchical topologies. Taking into consideration the properties of regular trees discussed previously, it is easy to understand that if u occupies a higher position than v in the hierarchy, then $\Delta_{uv} \geqslant 0$. Moreover, this parameter depends strongly on the value of the strength of the relationships. As Fig. 6 shows, when considering the domination of the top vertex with respect to any of its subordinates, a pronounced maximum appears in the region of low ξ (high strength of the relationships). This maximum can be calculated analytically using Eq. (6)–(7).

It can be proved that this result is very general and applicable to all levels of the hierarchy and to any regular tree. Intuitively, it is easy to understand that when $\xi = \infty$ then there is no information flow on the organization and $\Gamma_i = 1$ for all vertices, so applying Eq. (10) we may see that $\Delta_{uv} = 0$ independently of the position of u and v. On the other hand, when $\xi = 0$ there is no degradation of the information and then $\Gamma_i = N$, where N is the number of vertices in the graph and i is any of them, so $\Delta_{uv} = 0$ once more. As $\Delta_{uv} \geqslant 0$, then there must be at least one maximum (note that $\Delta_{uv} > 0$ for intermediate values of ξ).

6. Conclusions

Although the proposed model for propagation of information still needs some refinements [15], it is important to remark that such a simple model of human relationships is suitable to justify some important features of social networks. Moreover, the coordination degree and the average coordination degree, have proven to be suitable measurements of centrality and efficiency in social networks.

The results obtained in this paper appear to be of interest from a sociological perspective, because they dig on the foundations of social networks, showing that most organizations do not follow principles of global efficiency and effectiveness in the design of their topologies. But instead, the network growth is governed with the aim of maximizing the individual influence each actor has within the structure. We have proved that this kind of strategy drives to the development of networks with hierarchical structures. This provides an information centrality reflecting the position each actor occupies in the hierarchy, but has an important cost in terms of global efficiency.

We have also shown that hierarchical structures mainly benefit the higher levels by providing them a higher information centrality and improving their dominance of information, and this remains true even when the relationships on the network are strong. Moreover, there are some other drawbacks of hierarchical topologies that are out of the scope of this paper but that merit a mention here. The most remarkable one is that the betweenness centrality also follows hierarchical rules, meaning that the higher the position on the organization, the higher the power to intercept, manipulate and obscure information.

Furthermore, this paper is a reinforcement for the proposed model of information transfer and it is interesting for this field because it digs on the relation between the structure of complex networks and their function.

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